

Double Integral Change of Variables

Duo Tao

California Institute of Technology

January 19, 2019

What I want to do

Maybe you will laugh at me, but today I found I did not know how to change variables for a double integral properly. This website by Paul Dawkins taught me how to do it [1].

Say we want to integrate

$$\int fun(x, y) dx dy$$

We want to do the change of variables to u, v

$$x = f(u, v)$$

$$y = g(u, v)$$

The correct way

I won't bother telling you my wrong way. Here is the correct way.

$$dx dy = \begin{vmatrix} \frac{\partial f}{\partial u} & \frac{\partial f}{\partial v} \\ \frac{\partial g}{\partial u} & \frac{\partial g}{\partial v} \end{vmatrix} du dv \quad (1)$$

This is the most important part (where I was confused). The rest should be easy (change the integrand and the limits).

Example

A ubiquitous change: from x,y to r, θ .

$$x = r \cos \theta$$

$$y = r \sin \theta$$

Then plug in

$$dxdy = \begin{vmatrix} \cos \theta & -r \sin \theta \\ \sin \theta & r \cos \theta \end{vmatrix} drd\theta = r drd\theta \quad (2)$$

References



Paul Dawkins. *Calculus III - Change of Variables*.

<http://tutorial.math.lamar.edu/Classes/CalcIII/ChangeOfVariables.aspx>. Accessed: 2019-01-19.